



Fractions, Area/Perimeter, and Time  
Math in Focus

Unit 3 Curriculum Guide  
February 4<sup>th</sup>, 2019 – April 18<sup>th</sup>, 2019



ORANGE PUBLIC SCHOOLS  
OFFICE OF CURRICULUM AND INSTRUCTION  
OFFICE OF MATHEMATICS

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## Unit 3: Chapters 19, 14, 16

### Eureka Math Module 5

#### In this Unit Students will

- Develop an understanding of fractions, beginning with unit fractions.
- View fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole.
- Understand that the size of a fractional part is relative to the size of the whole. For example,  $\frac{1}{2}$  of the paint in a small bucket could be less paint than  $\frac{1}{3}$  of the paint in a larger bucket, but  $\frac{1}{3}$  of a ribbon is longer than  $\frac{1}{5}$  of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one.
- Solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
- Recognize that the numerator is the top number of a fraction and that it represents the number of equal sized parts of a set or whole; recognize that the denominator is the bottom number of a fraction and that it represents the total number of equal sized parts or the total number of objects of the set.
- Explain the concept that the larger the denominator, the smaller the size of the piece.
- Compare common fractions with like denominators and tell why one fraction is greater than, less than, or equal to the other.
- Represent halves, thirds, fourths, sixths, and eighths using various fraction models.
- Tell and write time to the nearest minute and measure time intervals in minutes.
- Solve word problems involving addition and subtraction of time intervals in minutes, e.g. by representing the problem on a number line diagram.

#### *Essential Questions ( Bold Writing= Largely Suggested)*

- How can fractions be represented?
- How does the denominator affect the size of the pieces?
- What do the denominator and numerator represent in a fraction?
- How can you compare unit fractions with same denominators?
- How can you compare fractions with the same numerator?
- How can you use visual models to compare simple equivalent fractions?
- What makes some fractions equivalent?
- How can fractions be represented on a number line?

### *Enduring Understandings*

- A fraction is a number.
- A fraction is a quantity when a whole is partitioned into equal parts.
- The whole that the fraction refers to must be specified.
- Unit fractions are the basic building blocks of fractions in the same way that 1 is the basic building block of whole numbers.
- Understand the concept of numerator and denominator.
- As the number of equal parts in the whole increases, the size of the fractional pieces decreases.
- The denominator represents the number of equal parts in the whole.
- The numerator is the count of the number of equal parts.
- Equivalent fractions represent the same size or the same point on a number line.
- When comparing fractions, each fraction must refer to the same whole.
- Fractions with common numerators or common denominators can be compared by reasoning about the number of parts or the size of the parts.
- Know fractions can represent parts of a whole, a point on a number line as well as distance on a number line.
- Understand that the size of a fractional part is relative to the size of the whole.
- Compare and order unit fractions.
- Compare and order fractions with like denominators.

## MIF Pacing Guide

Activity	Common Core Standards	Estimated Time (# of block)	Lesson Notes
19.1 Area	3.MD.5,6,7	1 block	
19.2 Square Units	3.MD.5,6,7	1 block	
19.3 Square Units	3.MD.5,6,7	1 block	
19.4 Perimeter and Area	3.MD.5,6,7, 3.MD.8 3.NBT.2	1 block	
19.5 More Perimeter	3.MD.8	1 block	
Problem Solving/ Chapter Wrap up	3.MD.5,6,7, 3.MD.8	1 block	
14.1 Understanding Fractions	3.NF.2, 3N.F.3	2 blocks	Have students count by fractions and highlight the different roles that the numerator and denominator have. Continually connect the vocabulary to models. . Read fractions with meaning. Example: $\frac{3}{4}$ reads, "3 out of 4 equal parts".
14.2 Understanding Equivalent Fractions	3.NF.1, 3N.F.2, 3.NF.3	2 blocks	
14.3 More Equivalent Fractions	3.NF.1, 3N.F.2, 3.NF.3	2 blocks	
14.4 Comparing Fractions	3.NF.2 and 3.NF.3	3 blocks	
Chapter 14 Wrap Up/Review	3.NF.1, 3N.F.2, 3.NF.3	1 block	Reinforce and consolidate chapter skills and concepts
Chapter 14 Test	3.NF.1, 3N.F.2, 3.NF.3	1 block	

## Eureka Math Module 5:

### Fractions as Numbers on the Number Line

Topic	Lesson	Lesson Objective/ Supportive Videos
<b>Topic D:</b> Fractions on the Number Line	Lesson 14	Place unit fractions on a number line with endpoints 0 and 1. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 15	Place any fraction on a number line with endpoints 0 and 1. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 16	Place whole number fractions and unit fractions between whole numbers on the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 17	Practice placing various fractions on the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 18	Compare fractions and whole numbers on the number line by reasoning about their distance from 0. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic E:</b> Equivalent Fractions	Lesson 20	Recognize and show that equivalent fractions have the same size, though not necessarily the same shape. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 21	Recognize and show that equivalent fractions refer to the same point on the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 22	Generate simple equivalent fractions by using visual fraction models and the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 23	Generate simple equivalent fractions by using visual fraction models and the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 24	Express whole numbers as fractions and recognize equivalence with different units. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 26	Decompose whole number fractions greater than 1 using whole number equivalence with various models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 27	Explain equivalence by manipulating units and reasoning about their size. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 28	Compare fractions with the same numerator pictorially. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>

<b>Topic F:</b> Comparison, Order, and Size of Fractions	Lesson 29	Compare fractions with the same numerator using $<$ , $>$ , or $=$ and use a model to reason about their size. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 30	Partition various wholes precisely into equal parts using a number line method. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>

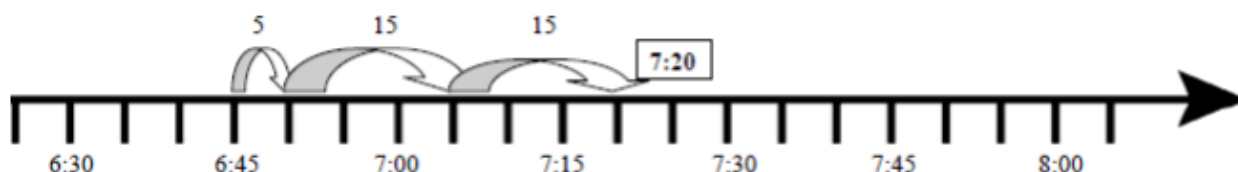
Activity	Common Core Standards	Time	Notes
16.1 Telling Time	3.MD.1	1 block	<b>Student must understand the units of time in order to successfully be able to add and subtract time.</b> Students may use open number lines or a drawing of an analog clock to add or subtract time.
16.2 Converting Hour and Minutes	3.MD.1	1 block	<u>Three Main Parts In Elapsed Time</u> <ul style="list-style-type: none"> <li>• Start Time</li> <li>• Time that Passed</li> <li>• End Time</li> </ul> Usually one of these parts is unknown
16.3 Adding Hours and Minutes	3.MD.1	1 block	
16.4 Subtracting Hours and Minutes	3.MD.1	1 block	
16.5 Elapsed Time	3.MD.1	3 blocks	
Chapter 16 Wrap Up/Review/ Test	3.MD.1	2 block	Reinforce and consolidate chapter skills and concepts

## Common Core State Standards

**3.MD.1**

Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g by representing the problem on a number line diagram.

- This standard calls for students to solve problems with elapsed time, including word problems. Students could use clock models or number lines to solve.
- Elapsed time is the time that has passed from one point to another. Finding elapsed time includes knowing the starting and ending time of an event, then determining how much time has passed.
- On the number line, students should be given the opportunities to determine the intervals and size of jumps. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).



- Students should use the number line as a visual model to solve real world problems involving time. Students should choose appropriate strategies to solve real world problems involving time.
- Model measurement vocabulary: *estimate, time, time intervals, minute, hour, and elapsed time.*

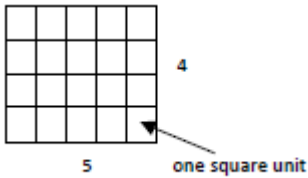
**3.MD.5**

Recognize area as an attribute of plane figures and understand concepts of area measurement.

- A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
- A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units.



These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper. Based on students' development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.



**3.MD.6**

Measure areas by counting unit squares (squares cm, square m, square in, square ft, and improvised units).

Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches. The task shown above would provide great experiences for students to tile a region and count the number of square units.

**3.MD.7**

Relate area to the operations of multiplication and addition.

- a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular area in mathematical reasoning.
- c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side length  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$ . Use area models to represent the distributive property in mathematical reasoning.
- d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real word problems.

Students can learn how to multiply length measurements to find the area of a rectangular region. But, in order that they make sense of these quantities, they must first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. This relies on the development of spatial

structuring. To build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows, students might draw rectangular arrays of squares and learn to determine the number of squares in each row with increasingly sophisticated strategies, such as skip-counting the number in each row and eventually multiplying the number in each row by the number of rows. They learn to partition a rectangle into identical squares by anticipating the final structure and forming the array by drawing line segments to form rows and columns. They use skip counting and multiplication to determine the number of squares in the array. Many activities that involve seeing and making arrays of squares to form a rectangle might be needed to build robust conceptions of a rectangular area structured into squares.

Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are.

Students should tile rectangle then multiply the side lengths to show it is the same.

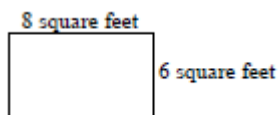
To find the area one could count the squares or multiply  $3 \times 4 = 12$ .

1	2	3	4
5	6	7	8
9	10	11	12

Students should solve real world and mathematical problems.

Example:

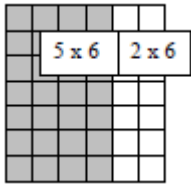
Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?



Students might solve problems such as finding all the rectangular regions with whole-number side lengths that have an area of 12 area units, doing this for larger rectangles (e.g. enclosing 24, 48, 72 area-units), making sketches rather than drawing each square.

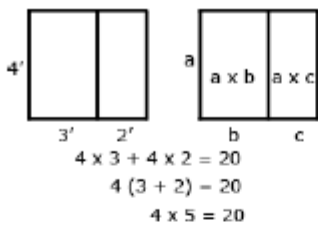
Students learn to justify their belief they have found all possible solutions.

This standard extends students' work with distributive property. For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.

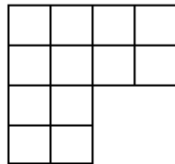


Using concrete objects or drawings students build competence with composition and decomposition of shapes, spatial structuring, and addition of area measurements, students learn to investigate arithmetic properties using area models. For example, they learn to rotate rectangular arrays physically and mentally, understanding that their area are preserved under rotation, and thus for example,  $4 \times 7 = 7 \times 4$ , illustrating the commutative property of multiplication. Students also learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying  $12 \times 5$ , or by adding two products, e.g.  $10 \times 5$  and  $2 \times 5$ , illustrating distributive property.

Example:



This standard uses the word *rectilinear*. A *rectilinear figure* is a polygon that has all right angles.



How could this figure be decomposed to help find the area?



This portion of the decomposed figure is  $4 \times 2$ .



This portion of the decomposed figure is  $2 \times 2$ .

$$4 \times 2 = 8 \text{ and } 2 \times 2 = 4$$

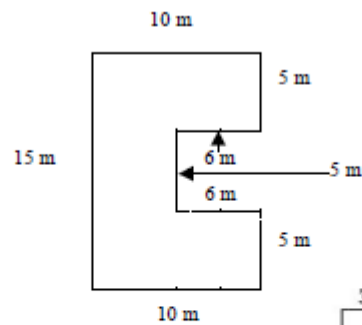
$$\text{So } 8 + 4 = 12$$

Therefore the total area of this figure is 12 square units

Example:

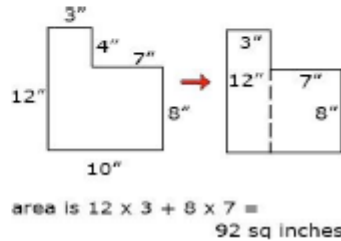
A storage shed is pictured below. What is the total area?

How could the figure be decomposed to help find the area?



Example:

Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



### 3.MD.8

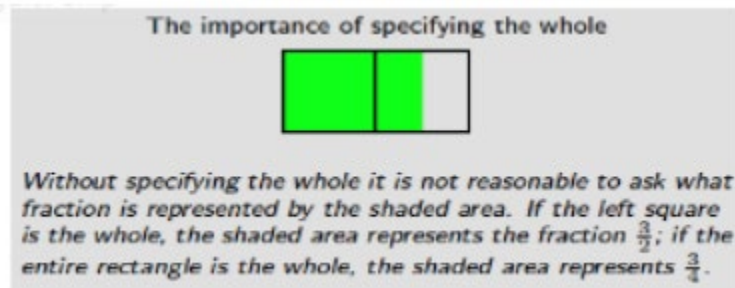
Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

- Measurement describes the attributes of objects and events. Standard units of measure enable people to interpret results or data
- Perimeter of a figure is equivalent to the sum of the length of all sides. Rectangles that have the same perimeter can have different areas. Rectangles that have same area can have different perimeters.
- Relate addition and subtraction to length.
- Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard.
- Show rectangles that have the same perimeter but different areas. Show rectangles having different perimeters but the same area.

### 3.NF.1

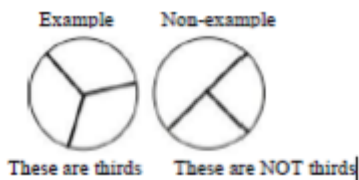
Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .

- This standard refers to the sharing of a whole being partitioned. Fraction models in third grade included only are (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a groups) are not addressed in Third Grade.
- In 3.NF. 1 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g. if a whole is partitioned into 4 equal parts then each part is  $\frac{1}{4}$  of the whole, and 4 copies of that part make the whole.
- Students build fractions from unit fractions, seeing the numerator 3 of  $\frac{3}{4}$  as saying that  $\frac{3}{4}$  is the quantity you get by putting 3 pieces of  $\frac{1}{4}$ 's together. There is no need to introduce "improper fraction" initially.



Some important concepts related to developing understanding of fractions include:

- Understand fractional parts must be equal-sized



- The number of equal parts tells how many parts make a whole.
- As the number of equal parts in the whole increases, the size of the fractional part decreases.
- The size of the fractional part is relative to the whole. One-half of a small pizza is

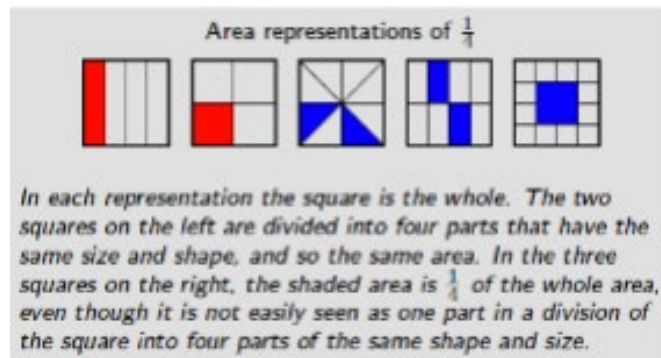
relatively smaller than one-half of a large pizza.

- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
- Students can count one fourth, two fourths, three fourths.
- Students express fractions as fair sharing or, parts of a whole. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require them to create and reason about fair share.
- Initially, students can use an intuitive notion of “same size and same shape” (congruence) to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for “equal parts” as “parts with equal measurements.”

**Example:**

When a ruler is partitioned into halves or quarters of an inch, students see that each subdivision has the same length.

In area models students reason about the area of a shaded region to decide what fraction of the whole it represents



**3.NF.2**

Understand a fraction as a number on the number line, represent fractions on a number line diagram.

- a. Represent a fraction  $1/b$  on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line.
- b. Represent a fraction  $a/b$  on a number line diagram by marking off a lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number.

- The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that  $\frac{1}{2}$  is between 0 and 1). Students need ample experiences folding linear models (e.g., strings, sentence strips) to help them reason about and justify the location of fractions, such that  $\frac{1}{2}$  lies exactly between 0 and 1.
- In the number line diagram, the space between 0 and 1 is divided (partitioned) into 4 equal parts. The distance from 0 to the first segments is 1 of the 4 parts from 0 to 1 or known as  $\frac{1}{4}$ . Similarly, the distance from 0 to the third segment is 3 parts that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction  $\frac{3}{4}$



**3.NF.3**

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- b. Recognize and generate simple equivalent fractions, e.g.  $\frac{1}{2} = \frac{2}{4}$ ,  $\frac{4}{6} = \frac{2}{3}$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form  $3 = \frac{3}{1}$ ; recognize that  $\frac{6}{1} = 6$ ; locate  $\frac{4}{4} = 1$  at the same point of a number line diagram.
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions e.g., by using visual fraction models.

- An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example,  $\frac{1}{8}$  is smaller than  $\frac{1}{2}$  because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 same size whole is cut into 2 pieces.
- 3.NF.3a and 3.NF.3b: These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures. This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction  $\frac{3}{1}$  is 3 wholes divided into one group.
- **This standard is the building block for later work where students divide set of objects into a specific number of groups. Students understand the meaning of  $\frac{a}{1}$**

**Example:**

If 6 brownies are shared between 2 people, how many brownies would each person get?

- 3.NF.d: This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason



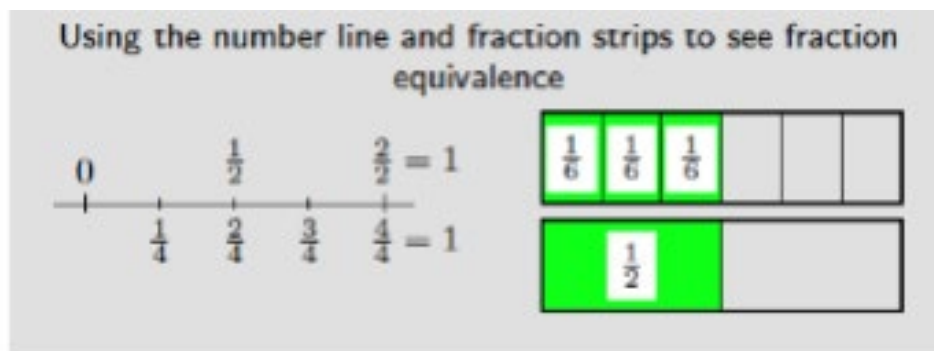
about the size of pieces, such as  $\frac{1}{3}$  of a cake being larger than  $\frac{1}{4}$  of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.

- In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example,  $\frac{1}{2}$  of a large pizza is a different amount than  $\frac{1}{2}$  of a small pizza. Students should be given opportunities to discuss and reason about which  $\frac{1}{2}$  is larger.
- Previously, in second grade, students compared lengths using a standard measure unit. In third grade, they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions.

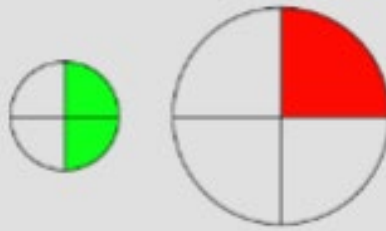
**Example:**

A segment from 0 to  $\frac{3}{4}$  is shorter than the segment from 0 to  $\frac{5}{4}$  because it measures 3 segments of  $\frac{1}{4}$  as opposed to 5 segments of  $\frac{1}{4}$ . Therefore,  $\frac{3}{4} < \frac{5}{4}$ .

- Students also see that unit fractions with a larger denominator are smaller, by reasoning that in order for more (identical) pieces to make the same whole, the pieces must be smaller.
- From this idea, they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example,  $\frac{2}{5} > \frac{2}{7}$ , because  $\frac{1}{7} < \frac{1}{5}$ , so 2 pieces of  $\frac{1}{7}$  is less than 2 pieces of  $\frac{1}{5}$ . As with equivalence of fractions, it is important to make sure that each fraction refers to the same whole when comparing fractions.



The importance of referring to the same whole when comparing fractions



*A student might think that  $\frac{1}{4} > \frac{1}{2}$ , because a fourth of the pizza on the right is bigger than a half of the pizza on the left.*

# MIF Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	<p><b>Chapter Opener</b> Assessing Prior Knowledge</p> <p><i>The Pre Test serves as a diagnostic test of readiness of the upcoming chapter</i></p>	<p><b>Teacher Materials</b> Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge</p> <p><b>Student Materials</b> Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge</p>	<p>Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding</p> <p>Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper</p> <p>Quick Check and the Pre Test can be done in the same block (<i>See Anecdotal Checklist; Transition Guide</i>)</p> <p>Recall Prior Knowledge – Quick Check – Pre Test</p>
DIRECT ENGAGEMENT	<p><b>Direct Involvement/Engagement</b> Teach/Learn</p> <p><i>Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions</i></p>	<p><b>Teacher Edition</b> 5-minute warm up Teach; Anchor Task</p> <p><b>Technology</b> Digi</p> <p><b>Other</b> Fluency Practice</p>	<ul style="list-style-type: none"> <li>• The Warm Up activates prior knowledge for each new lesson</li> <li>• Student Books are CLOSED; Big Book is used in Gr. K</li> <li>• Teacher led; Whole group</li> <li>• Students use concrete manipulatives to explore concepts</li> <li>• A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning</li> <li>• Teacher facilitates; Students find the solution</li> </ul>
GUIDED LEARNING	<p><b>Guided Learning and Practice</b> Guided Learning</p>	<p><b>Teacher Edition</b> Learn</p> <p><b>Technology</b> Digi</p> <p><b>Student Book</b> Guided Learning Pages Hands-on Activity</p>	<p>Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking</p> <p><b>Small Group w/Teacher circulating among groups</b> Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time</p>

INDEPENDENT PRACTICE	<p><b>Independent Practice</b></p> <p><i>A formal formative assessment</i></p>	<p><b>Teacher Edition</b> Let's Practice</p> <p><b>Student Book</b> Let's Practice</p> <p><b>Differentiation Options</b> All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment</p>	<p><b>Let's Practice</b> determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice.</p> <p>Manipulatives <b>CAN</b> be used as a communications tool as needed.</p> <p>Completely Independent</p> <p>On level/advance learners should finish all workbook pages.</p>
ADDITIONAL PRACTICE	<p><b>Extending the Lesson</b></p>	<p>Math Journal Problem of the Lesson Interactivities Games</p>	
	<p><b>Lesson Wrap Up</b></p>	<p>Problem of the Lesson</p> <p>Homework (Workbook, Reteach, or Extra Practice)</p>	<p>Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice)</p> <p>Reteach Homework (issued to struggling learners) should be checked the next day</p>
POST TEST	<p><b>End of Chapter Wrap Up and Post Test</b></p>	<p><b>Teacher Edition</b> Chapter Review/Test Put on Your Thinking Cap</p> <p><b>Student Workbook</b> Put on Your Thinking Cap</p> <p><b>Assessment Book</b> Test Prep</p>	<p>Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is <u>graded/scored</u>.</p> <p>The Chapter Review/Test can be completed</p> <ul style="list-style-type: none"> <li>Individually (e.g. for homework) then reviewed in class</li> <li>As a 'mock test' done in class and doesn't count</li> <li>As a formal, in class review where teacher walks students through the questions</li> </ul> <p>Test Prep is completely independent; scored/graded</p> <p>Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.</p>

## Math Background

During their elementary mathematics education, third-grade students will have prior knowledge/experience related to the concepts and skills identified in this unit. In first grade, students are expected to partition circles and rectangles into two or four equal shares, and use the words, halves, half of, a fourth of, and quarter of. In second grade, students are expected to partition circles and rectangles into two, three, or four equal shares, and use the words, halves, thirds, half of, a third of, fourth of, quarter of. Students should also understand that decomposing into more equal shares equals smaller shares, and that equal shares of identical wholes need not have the same shape.

Students learned to read time from an analog clock to the hour and half hour, and relate time to daily activities using terms such as o'clock and half past. Students learned about the minute hand, and the fact that time after the hour can be read in skips of 5 minutes. They learned the digital notation of time and the use of a.m. for time from midnight and p.m. for the time from noon.

## Misconceptions

- Some third graders may have difficulty simply reading a clock to tell time. Before teaching elapsed time. Make sure students can tell time to the minute. Allow students to use a clock with movable hands, but keep in mind that numerous ongoing practices telling time to the minute using a clock or number line to show elapsed time will help students become proficient.
- Students may incorrectly miscount the unit squares covered to determine the area of a shape using graph paper. To avoid an incorrect count, students can put the numbers of the counting sequences in each square as they count them.
- Some students may be challenged by simply visualizing and finding the rectangles in the figures.
- Students are often confused between the concepts of perimeter and area. To address this, provide additional experience for students to discover that the concept of an object's perimeter as a one-dimensional attribute and area as two-dimensional.
- Do not work with too many representations as the same time. Begin with activities that use area models and reinforce those ideas of fraction strips and then number lines. **For most students one experience with a concept will not be adequate to develop deep understanding.**
- Although it is not critical for students to differentiate between the intervals between points and actual points on the number line, you want to be careful not to cause any misconceptions.
- As students work with equivalent fractions, it is important that they understand that different fractions can name the same quantity and there is a multiplicative relationship between equivalent fractions.
- **The following misconceptions indicate the students need more work with concrete and pictorial representations:**
  - *The numerator cannot be greater than the denominator*
  - *The larger the denominator, the larger the piece*
  - *Fractions are a part of a whole*
  - *In building sets of equivalent fractions, students use addition or subtraction to find equivalent fractions.*

## PARCC Assessment Evidence/Clarification Statements

NJSLs	Evidence Statement	Clarification	Math Practices
3.MD.1-1	Tell and write time to the nearest minute and measure time intervals in minutes.	<ul style="list-style-type: none"> <li>• Time intervals are limited to 60 minutes</li> <li>• No more than 20% of items require determining a time interval from clock readings having different hour values.</li> <li>• Acceptable interval: Start time 1:20, end time 2:10 – time interval is 50 minutes. Unacceptable interval: Start time 1:20, end time 2:30 – time interval exceeds 60 minutes.</li> </ul>	
3.MD.1-2	Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	<ul style="list-style-type: none"> <li>• Only the answer is required.</li> <li>• Tasks do not involve reading start/stop times from a clock nor calculating elapsed time</li> </ul>	MP.1, MP.2, MP.4, MP.5
3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. <ol style="list-style-type: none"> <li>a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.</li> <li>b. A plane figure which can be covered without gaps or overlaps by <math>n</math> unit squares is said to have an area of <math>n</math> square units.</li> </ol>	None	MP.7
3.MD.6	Measure areas by counting unit squares (squares cm, square m, square in, square ft, and improvised units).	None	MP.7
	Solve real world and mathematical problems involving perimeters of		MP.2,4,5

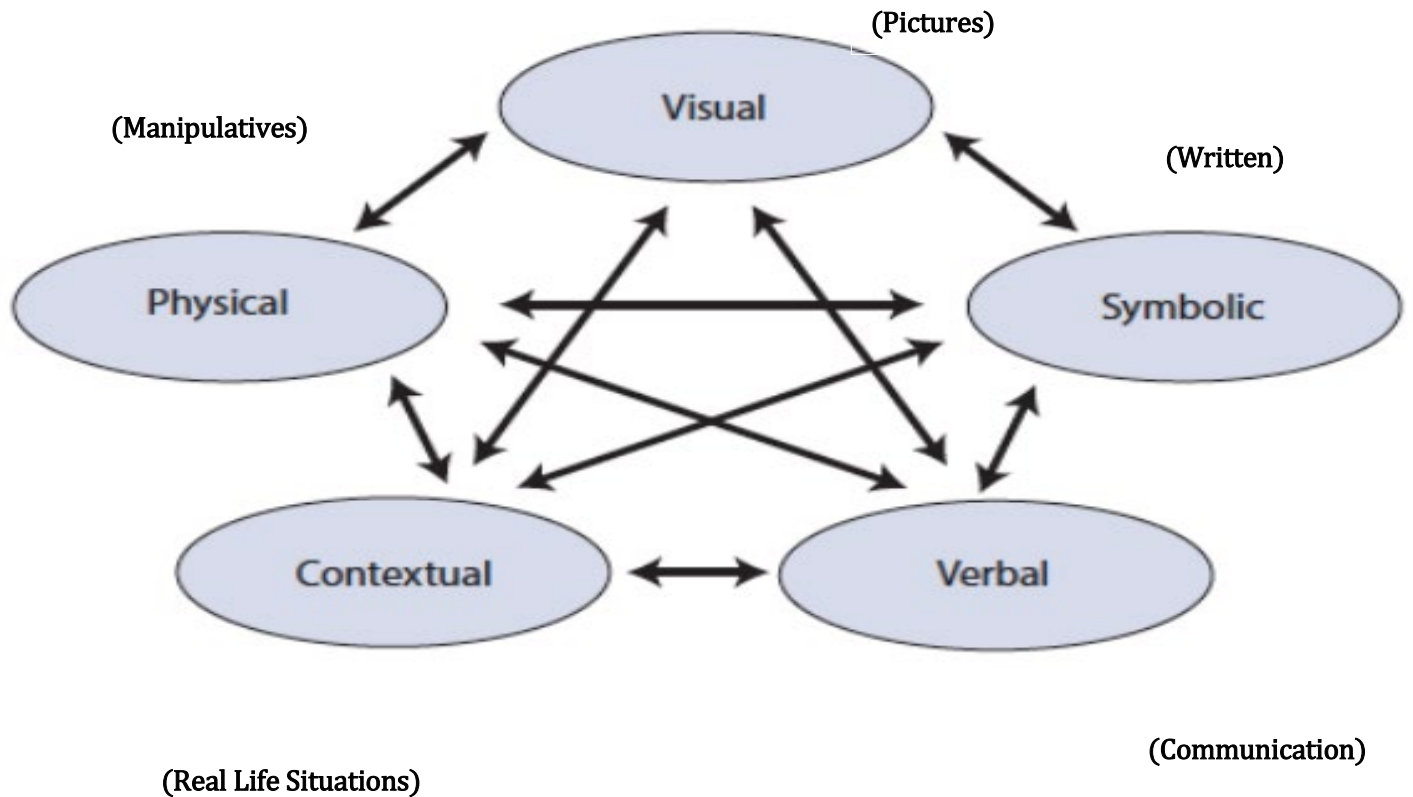
3.MD.8	polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.		
3.NF.1	Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$ .	<ul style="list-style-type: none"> <li>• Tasks do not involve the number line.</li> <li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8</li> </ul>	MP 2
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. b. Represent a fraction $a/b$ on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number $a/b$ on the number line.	<ul style="list-style-type: none"> <li>• Fractions may be greater than 1.</li> <li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>• Fractions equal whole numbers in 20% of these tasks.</li> <li>• Tasks have “thin context”<sup>2</sup> or no context.</li> <li>• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> </ul>	MP 5
3.NF.3a-1	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size	<ul style="list-style-type: none"> <li>• Tasks do not involve the number line.</li> <li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>• The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 5



3.NF.3a-2	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same point on a number line	<ul style="list-style-type: none"> <li>• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>• The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 5
3.NF.3b-1	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$ , $4/6 = 2/3$ .	<ul style="list-style-type: none"> <li>• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>• The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 7
3.NF.3c	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$ ; recognize that $6/1 = 6$ ; locate $4/4$ and 1 at the same point of a number line diagram.	<ul style="list-style-type: none"> <li>• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>• The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 3, 5, 7
3.NF.3d	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$ , $=$ , or	Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Justifying is not assessed here. For this aspect of 3.NF.3d, see 3.C.3-1 and 3.C.4-4. iv) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP 7

3.NF.A.Int. 1	In a contextual situation involving a whole number and two fractions not equal to a whole number, represent all three numbers on a number line diagram, then choose the fraction closest in value to the whole number.	<ul style="list-style-type: none"><li>• Fractions equivalent to whole numbers are limited to 0 through 5.</li><li>• Fraction denominators are limited to 2, 3, 4, 6 and 8.</li></ul>	MP 2,4,5
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## Use and Connection of Mathematical Representations



**The Lesh Translation Model**

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical:** The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal:** Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

**Symbolic:** Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

### **The Lesh Translation Model: Importance of Connections**

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

## Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** “Doing Stage”: Physical manipulation of objects to solve math problems.

**Pictorial:** “Seeing Stage”: Use of imaged to represent objects when solving math problems.

**Abstract:** “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

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## Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

## Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

### Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?

The most  
important thing  
is to NEVER  
stop  
questioning

*Albert Einstein*

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

**100** questions that promote  
**Mathematical Discourse**

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** \_\_\_?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** \_\_\_ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is **mathematically correct**

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

## Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** \_\_\_\_\_?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

## Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

## Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?





## Help students learn to conjecture, invent, and solve problems

- 48 What would happen if \_\_\_?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram or make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.



## Help students learn to connect mathematics, its ideas, and its application

- 74 What is the **relationship** between \_\_\_ and \_\_\_?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?
- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to \_\_\_?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

### Help students persevere

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?
- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

### Help students focus on the mathematics from activities

## **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

## **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

## **Math Fact Fluency: Automaticity**

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the [mind](#) with the low-level details required, allowing it to become an automatic response pattern or [habit](#). It is usually the result of [learning](#), [repetition](#), and practice.

### **3-5 Math Fact Fluency Expectation**

**3.OA.C.7:** Single-digit products and quotients (Products from memory by end of Grade 3)

**3.NBT.A.2:** Add/subtract within 1000

**4.NBT.B.4:** Add/subtract within 1,000,000/ Use of Standard Algorithm

**5.NBT.B.5:** Multi-digit multiplication/ Use of Standard Algorithm

## Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

## Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

## Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

*Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*

(William 2007, pp. 1054; 1091)

## Connections to the Mathematical Practices

### **Student Friendly Connections to the Mathematical Practices**

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

### Connections to the Mathematical Practices

<b>1</b>	<p><b>Make sense of problems and persevere in solving them</b></p> <p>In <b>third</b> grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try approaches. They often will use another method to check their answers.</p>
<b>2</b>	<p><b>Reason abstractly and quantitatively</b></p> <p>In <b>third</b> grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities</p>
<b>3</b>	<p><b>Construct viable arguments and critique the reasoning of others</b></p> <p>In <b>third</b> grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is it true?” They explain their thinking to others and respond to others’ thinking.</p>
<b>4</b>	<p><b>Model with mathematics</b></p> <p>Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc...Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. <b>Third</b> graders should evaluate their results in the context of the situation and reflect whether the results make any sense.</p>
<b>5</b>	<p><b>Use appropriate tools strategically</b></p> <p><b>Third</b> graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities into an organized list or a table, and determine whether they all have the possible rectangles.</p>
<b>6</b>	<p><b>Attend to precision</b></p> <p>Mathematical proficient <b>third</b> graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.</p>
<b>7</b>	<p><b>Look for and make use of structure</b></p>

	In <b>third</b> grade, students should look closely to discover a pattern of structure. For example, students' properties of operations as strategies to multiply and divide. (commutative and distributive properties).
<b>8</b>	<b>Look for and express regularity in repeated reasoning</b>
	Mathematically proficient students in third <b>grade</b> should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of $7 \times 8$ , they might decompose 7 into 5 and 2 and then multiply $5 \times 8$ and $2 \times 8$ to arrive at $40 + 16$ or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

## Effective Mathematics Teaching Practices

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.



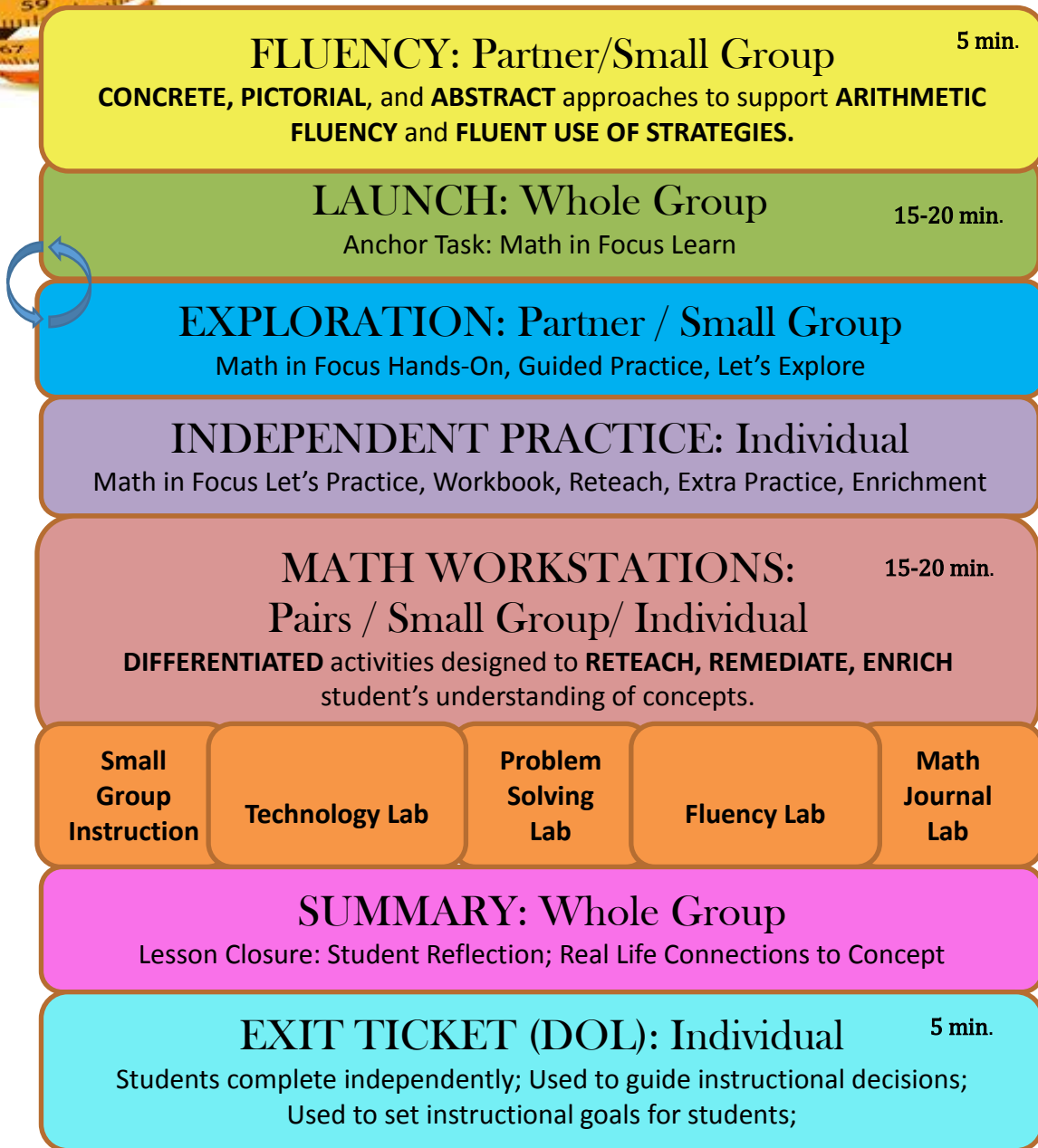
## **5 Practices for Orchestrating Productive Mathematics Discussions**

Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>



# 3<sup>rd</sup> and 4<sup>th</sup> Grade Ideal Math Block

## Essential Components



### Note:

- Place emphasis on the flow of the lesson in order to ensure the development of students' conceptual understanding.
- Outline each essential component within lesson plans.
- Math Workstations may be conducted in the beginning of the block in order to utilize additional support staff.
- Recommended: 5-10 technology devices for use within **TECHNOLOGY** and **FLUENCY** workstations.

### Unit 3 Assessment / Authentic Assessment Framework

Assessment	NJSLS	Estimated Time	Format	Graded
Chapter 19				
Optional Chapter 19 Test/Performance Task	3.MD.5-8	1 block	Individual	Yes
Authentic Assessment : Area of the Pool	3.MD.C.7.D	½block	Individual	Yes
Chapter 14				
Optional Chapter 14 Test/Performance Task	3.NF.1-3	1 block	Individual	Yes
Authentic Assessment: Equivalent Fractions	3.NF.3	½ block	Individual	Yes
Eureka Module 5				
End of Module Assessment	3.NF.1-3	1 block	Individual	Yes
Chapter 16				
Optional Chapter 16 Test/Performance Task	3.MD.1	1 block	Individual	Yes

	PLD	Genesis Conversion
<b>Rubric Scoring</b>	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

## Area of the Pool

The Hernandez family wants to install has an “L” shaped pool in their backyard that has an area of 54 square meters. What could the pool look like?

Draw the pool. Label the lengths of each side.

**3.MD.C.7.D:** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Students who demonstrate full accomplishment might divide the shape into two rectangles and accurately find the area of each figure by selecting a strategy such multiplying using arrays, counting the squares or adding the rows/columns of shaded squares and then adding the areas of the two rectangles together to get the total area.

Students who demonstrate partial accomplishment may confuse area and perimeter. Or students who demonstrate partial accomplishment may try to find the area of the total figure without dividing it into two rectangles, which could result in an incorrect answer.

<b>Level 5: Distinguished Command</b>	<b>Level 4: Strong Command</b>	<b>Level 3: Moderate Command</b>	<b>Level 2: Partial Command</b>	<b>Level 1: No Command</b>
<p>Clearly constructs and communicates a complete response based on explanations/reasoning using (the):</p> <ul style="list-style-type: none"> <li>• Decomposition of the shape into separate rectangles</li> <li>• Finding the area of each rectangle and then adding them together</li> <li>• Units of measurement</li> </ul> <p>Response includes an <b>efficient</b> and logical progression of steps.</p>	<p>Clearly constructs and communicates a complete response based on explanations/reasoning using (the):</p> <ul style="list-style-type: none"> <li>• Decomposition of the shape into separate rectangles</li> <li>• Finding the area of each rectangle and then adding them together</li> <li>• Units of measurement</li> </ul> <p>Response includes a <b>logical</b> progression of steps</p>	<p>Clearly constructs and communicates a complete response based on explanations/reasoning using (the):</p> <ul style="list-style-type: none"> <li>• Decomposition of the shape into separate rectangles</li> <li>• Finding the area of each rectangle and then adding them together</li> <li>• Units of measurement</li> </ul> <p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p>Clearly constructs and communicates a complete response based on explanations/reasoning using (the):</p> <ul style="list-style-type: none"> <li>• Decomposition of the shape into separate rectangles</li> <li>• Finding the area of each rectangle and then adding them together</li> <li>• Units of measurement</li> </ul> <p>Response includes an <b>incomplete or Illogical</b> progression of steps.</p>	<p>The student shows no work or justification</p>

Mrs. Caha asked her class to write fractions on their whiteboards that were equivalent to  $\frac{1}{2}$ . Tell if each student's fraction is equivalent to Mrs. Caha's fraction and show how you know.

Gloria : $\frac{3}{4}$	CIRCLE ONE  <input type="radio"/> Yes <input type="radio"/> No	Show how you know:
Isaiah: $\frac{2}{3}$	CIRCLE ONE  <input type="radio"/> Yes <input type="radio"/> No	Show how you know:
Thomas: $\frac{4}{8}$	CIRCLE ONE  <input type="radio"/> Yes <input type="radio"/> No	Show how you know:

## Authentic Assessment Scoring Rubric – Equivalent Fractions

**3.NF.A.3:** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

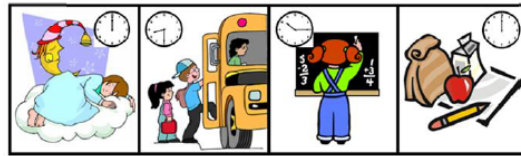
Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>Number line, visual model, reasoning about size</li> </ul> <p>Response includes an <b>efficient</b> and logical progression of steps.</p>	<p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>Number line, visual model, reasoning about size</li> </ul> <p>Response includes a <b>logical</b> progression of steps</p>	<p>Constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>Number line, visual model, reasoning about size</li> </ul> <p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p>Constructs and communicates an incomplete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>Number line, visual model, reasoning about size</li> </ul> <p>Response includes an <b>incomplete or illogical</b> progression of steps.</p>	<p>The student shows no work or justification</p>

**Visual Definition**

**The terms below are for teacher reference only and are not to be memorized by students.**

Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

**a.m.**



12:00 A.M. 12 midnight      8:30 A.M. half past 8      10:15 A.M. a quarter after 10      12:00 P.M. noon

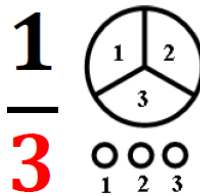
A time between 12:00 midnight and 12:00 noon.

**analog clock**



A clock that shows the time by the positions of the hour and minute hand.

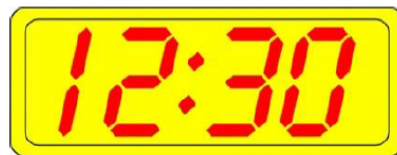
**denominator**



- Parts in all
- Whole
- Set
- Total

The quantity below the line in a fraction. It tells how many equal parts are in the whole.

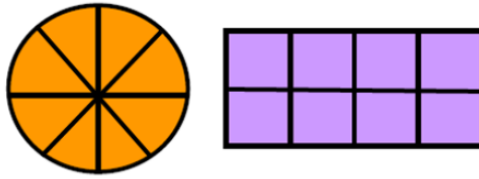
**digital clock**



A clock that shows the time with numbers of hours and minutes, usually separated with a colon. (:)



**eighths**



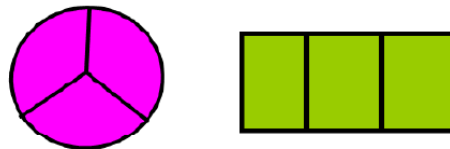
The parts you get when you divide something into eight equal parts.

**elapsed time**



The amount of time that has passed. (also known as time interval)

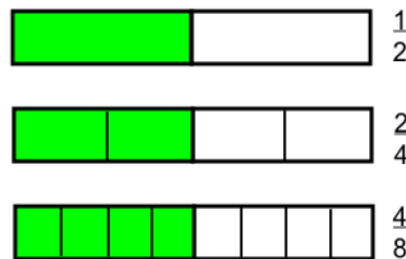
**equal parts**



3 equal parts

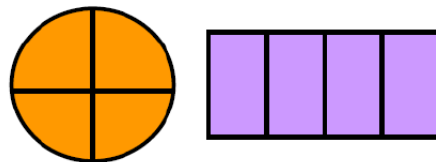
Parts of an object or group that have been divided equally into pieces.

**equivalent fractions**



Fractions that have the same value.

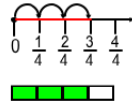
**fourths**



The parts you get when you divide something into 4 equal parts.

# fraction

Measurement Model



Bar Diagram  
(thickened number line)

Set Model

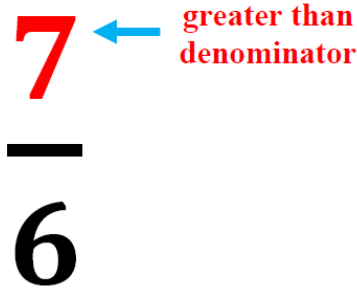


Area Model



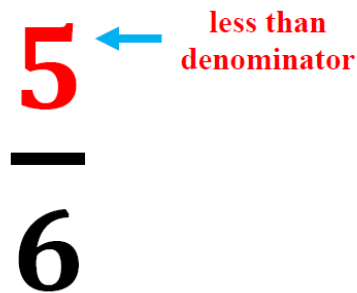
A way to describe a part of a whole or a part of a group by using equal parts.

# fraction greater than one



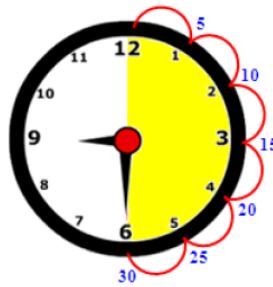
A fraction with the numerator greater than the denominator.

# fraction less than one



A fraction with the numerator less than the denominator.

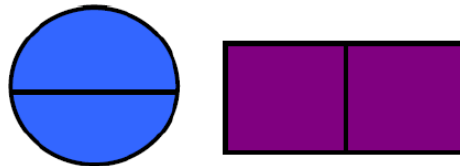
# half hour



30 minutes = one half-hour

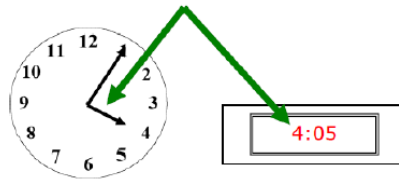
A unit of time equal to 30 minutes.

# halves



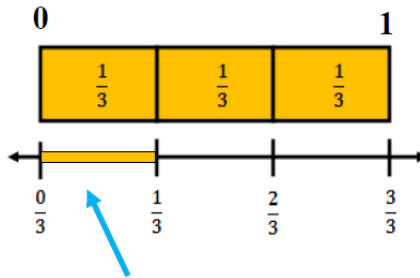
The parts you get when you divide something into 2 equal parts.

# hour (hr)



Units of time.  
1 hour = 60 minutes  
24 hours = 1 day

# interval



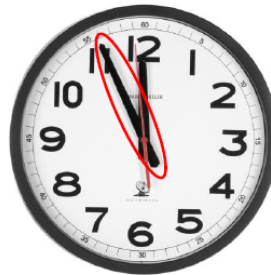
The distance between two points.

# midnight



12:00 at night.

# minute (min)



A unit used to measure short amounts of time; there are 60 minutes in one hour.

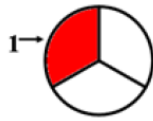
# noon



12:00 in the day.

**numerator**

$$\frac{1}{3}$$



- Parts shaded
- Parts we are using

The number written above the line in a fraction. It tells how many equal parts are described in the fraction.

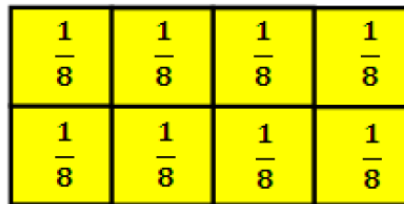
**order**

$$\frac{2}{8} \quad \frac{2}{6} \quad \frac{2}{4}$$

A sequence or arrangement of things. To order fractions, compare two fractions at a time.

In order from least to greatest.

**partition**



An action to divide shapes into smaller parts.

eight  $\frac{1}{8}$  equal parts

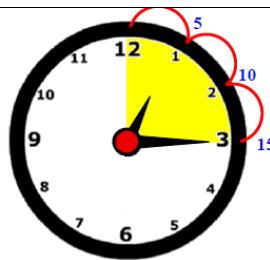
**p.m.**



12:00 P.M. noon      3:30 P.M. half past 3      7:45 P.M. a quarter to 8      12:00 A.M. 12 midnight

The time between 12:00 noon and 12:00 midnight.

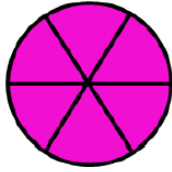
**quarter hour**



A unit of time worth 15 minutes.

15 minutes = 1 quarter hour

**sixths**



The parts you get when you divide something into six equal parts.

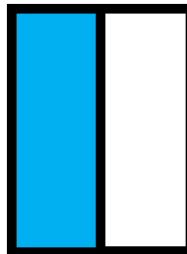
**thirds**



The parts you get when you divide something into 3 equal parts.

**unit  
fraction**

$$\frac{1}{2}$$



Example

A fraction that has 1 as its numerator.  
A unit fraction names 1 equal part of a whole.

**whole**



1 whole pie



1 whole rectangle

All of an object, a group of objects, shape, or quantity.

## 21<sup>st</sup> Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.

CRP2. Apply appropriate academic and technical skills.

CRP3. Attend to personal health and financial well-being.

CRP4. Communicate clearly and effectively and with reason.

CRP5. Consider the environmental, social and economic impacts of decisions.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.

CRP10. Plan education and career paths aligned to personal goals.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.